Infinite Series

Monday, April 10, 2023 8:55 AM

recall: approximate f(x) (goal) by better & better polynomials (higher & higher degrees)

$$e^x \overset{x\approx 0}{\approx} \underbrace{1 + x}_{\deg 1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

(1 = deg 0)

* factorials = product of #'s 1 up to n *

$e^0 = 1$

$$e^{0.1} \approx 1 + 0.1 + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{6} + \frac{(0.1)^4}{24} + \frac{(0.1)^5}{120} + \dots \Big\} \; 1.1051709$$

($e^{0.1} = 1.105171$; $\frac{(0.1)^2}{2} = 0.005$, $\frac{(0.1)^3}{6} = 0.0001\overline{6}$)

challenge: right hand side sums ∞ many #'s !!!

- we have sequence

$a_1 = 1$

$a_2 = 0.1$

$a_3 = \frac{(0.1)^2}{2}$

$a_4 = \frac{(0.1)^3}{6}$

⋮

use sequence → build new sequence:

$s_1 = a_1 = 1$

$s_2 = a_1 + a_2 = 1.1$

$s_3 = a_1 + a_2 + a_3 = 1.105$

$s_4 = a_1 + a_2 + a_3 + a_4$

better & better approximation (partial sums)

def: given sequence (a_n), its sequence of PARTIAL SUMS is defined by (s_m) (series) with $s_m := a_1 + \dots + a_m$

* series = sum of parts of sequence *

notation: $\sum_{n=1}^{m} a_n := a_1 + a_2 + \dots + a_m$

the limit is denoted (if exists): $\boxed{\lim_{m\to\infty} s_m := \sum_{n=1}^{\infty} a_n}$

* only do if a_n converges *

ex) $a_n = \frac{1}{2^n}$, looks like $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots$

partial sums:

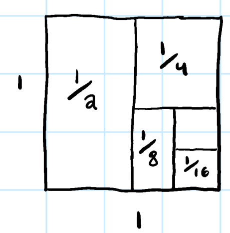
$s_1 = a_1 = \frac{1}{2} = 0.5$

$s_2 = a_1 + a_2 = \frac{1}{2} + \frac{1}{4} = 0.75$

$s_3 = a_1 + a_2 + a_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875$

$s_4 = a_1 + a_2 + a_3 + a_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 0.9375$

- s_m converges
- limit is $\boxed{\sum_{n=1}^{\infty} \frac{1}{2^n} = 1}$



sum = 1

ex of matters going bad:

1) $a_n = 3^n \longrightarrow \sum_{n=1}^{\infty} 3^n = 3 + 9 + 27 + \dots$ (a_1, a_2, a_3)

↑ if partial sums diverges: doesn't make sense

$s_1 = 3$

$s_2 = 12$

$s_3 = 39$

⋮

} diverges (goes to ∞)

2) $a_n = (-1)^n \longrightarrow \sum_{n=1}^{\infty} (-1)^n = -1 + 1 + -1 + -1 + 1 \dots$

(doesn't make sense)

$s_1 = -1$

$s_2 = 0$

$s_3 = -1$

$s_4 = 0$

⋮

} diverges (oscillating)